Microwave Radiation from Ferrimagnetically Coupled Electrons in Transient Magnetic Fields*

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Summary—Under certain restrictive conditions it appears that ferrimagnetically coupled electron spins are capable of coupling energy from a transient magnetic field and giving it up in the form of microwave radiation.

This paper analyzes the behavior of the uniform precession of motion in ferrimagnetic insulators under the influence of transient magnetic fields of changing amplitude and direction.

The expected radiation power and efficiency are calculated for such an oscillator employing yttrium iron garnet.

Introduction

THE transient behavior of ferrimagnetically coupled electrons in pulsed magnetic fields is of considerable interest since it has been found that under certain conditions such electrons appear capable of coupling energy from the field and converting it into electromagnetic radiation. R. V. Pound of Harvard University has proposed a solid-state ferrimagnetic microwave oscillator that would operate as follows.

A small ferrimagnetic crystal is placed in the minimum constant magnetic field required for saturation. At right angles to this constant field, a large pulsed magnetic field is applied. The electrons previously aligned to the constant biasing field will now align themselves with the resultant. If the rise time of the pulsed field is sufficiently rapid, this realignment will consist of the electrons precessing about the larger field, gradually spiraling in at a rate dependent upon the damping. This damping is due to radiation from the precession and heat losses. To be practical, the pulse magnetic rise time must be fast compared with the relaxation time constant of the ferrimagnetic crystal and the latter large enough for an appreciable pulse of radiation to result. The frequency of the radiated energy depends upon the precession frequency of the electrons which is a function of the magnitude of the magnetic pulse as given by the Larmor relation $\omega = -\gamma\mu_0 H$ where γ is the gyromagnetic ratio (including g factor) for ferromagnetic electrons.

In a private conversation with the author, C. L. Hogan postulated that the rise time of the pulsed magnetic field would be critical in another sense as illustrated in Fig. 1.

Here the magnetic field is assumed to jump in discrete steps (as from H_1 to H_2) during the rise. Originally, the electron magnetic moment vector precesses about H_1, generating a cone of small angle. If the jump of the magnetic field to H_2 occurs when the tip of the vector is at x, the small circle is the new precession path; if the tip of the vector is at y, the large circle is the precession path. It is extremely critical in this example when the magnetic field jump occurs, for in the first case the electron will have coupled energy *to* the magnetic field whereas in the second case it will have coupled energy *from* the field. The behavior is not quite so obvious when the discrete jumps are made continuous and so a theoretical analysis was undertaken. The formulation is for an electron interacting with a transient magnetic field of changing amplitude and direction but always lying in the same plane.

The problem can be generalized for the field moving in three dimensions but this is a needless complexity for this application.

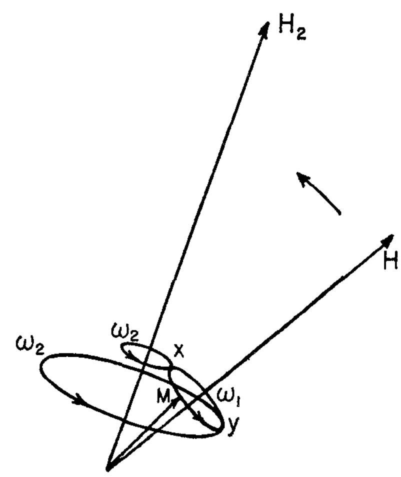

Fig. 1—Magnetization under transient field conditions.

Differential Equations

A Lagrangian formulation is used to derive the differential equations of motion of the magnetization. Fig. 2 gives the coordinate systems to be used. A spherical sample shape is assumed, so that the Kittel frequency will be independent of θ. Since demagnetizing effects cancel, only a single classical electron spin needs to be considered. The electron is considered to be a sphere of mass M, containing a charge e, located at the origin. It is spinning about a diameter with some radian frequency ω_0, with an angular momentum J_0, and magnetic moment $\boldsymbol{m} = \gamma J_0$ (where $\gamma \simeq -e/M$). The mks system of units will be used throughout.

Reference to the figure shows that $x'y'z'$ is a moving

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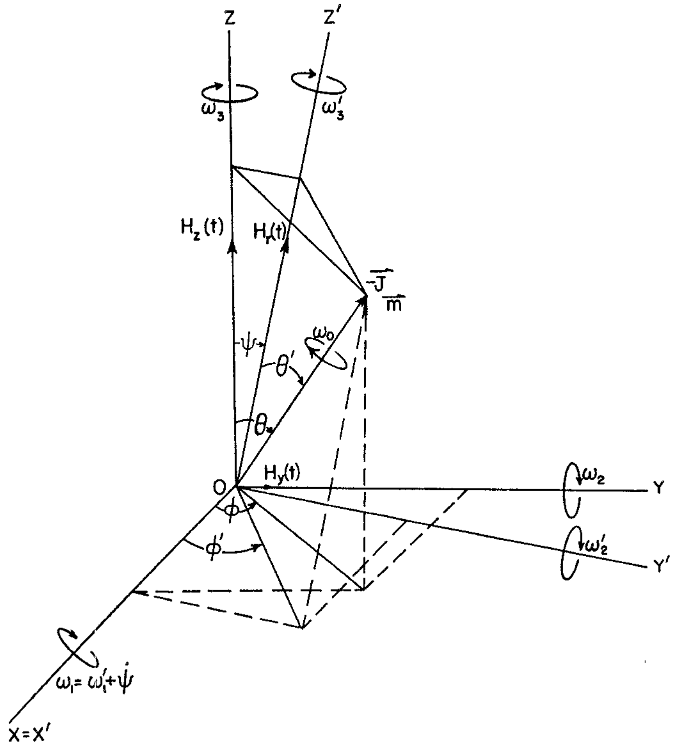

Fig. 2—Rotating coordinate system.

coordinate system which rotates so as to have its z' axis aligned with the instantaneous resultant internal magnetic field $H_r(t)$. This resultant is assumed always to stay in the yz plane of the fixed coordinate system xyz so that the angle $\psi(t)$ between z and z' is also in this plane.

The total kinetic energy of the electron written in terms of the moving coordinate system is

$$T = \frac{I_0}{2}\left[(\dot\theta')^2 + (\dot\phi')^2 + \omega_0{}^2 + (\dot\psi)^2 - 2\omega_0\dot\phi'\cos\theta' + 2\dot\psi(\dot\theta'\sin\phi' + \omega_0\cos\phi'\sin\theta')\right] \quad (1)$$

where I_0 is the moment of inertia. The potential energy $(\boldsymbol{m}\cdot\mu_0\boldsymbol{H})$ is given by

$$V = \mu_0 m H_r(t)(1 - \cos\theta'). \quad (2)$$

The Langrangian function is $L = T - V$. The differential equations are given by

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot q_i}\right) - \frac{\partial L}{\partial q_i} = Q_i \qquad i = 1, 2$$

$$q_1 = \theta' \qquad q_2 = \phi$$

where Q_i is a dissipation term.

It is convenient to break the pulsed magnetic field transient into two sections; first a buildup and second a constant plateau and decay. The pulses would in practice be periodic but are assumed to be separated far enough in time so that only one need be considered. The rise time of the pulse, T, will be assumed short compared with the relaxation times involved and so during the pulse buildup when ψ is changing the system will be assumed conservative and $Q_i = 0$.

The differential equations then become (since $\gamma = -m/I_0\omega_0$)

$$\frac{\ddot\theta'}{\omega_0} - \dot\phi'\sin\theta' + \frac{\ddot\psi}{\omega_0}\sin\phi' + \dot\psi\cos\phi'\left(\frac{\dot\phi'}{\omega_0} - \cos\theta'\right) - \mu_0\gamma H_r(t)\sin\theta' = 0 \quad (3)$$

$$\frac{\ddot\phi'}{\omega_0} - \dot\theta'\sin\theta' - \dot\psi\left(\frac{\dot\theta}{\omega_0}\cos\phi' - \sin\theta'\sin\phi'\right) = 0. \quad (4)$$

Since ω_0 is very large compared with the various derivatives it is permissible to approximate the equations as

$$\dot\phi' + \dot\psi\cos\phi'\cot\theta + \mu_0\gamma H_r(t) = 0 \quad (5)$$

$$\dot\theta' + \dot\psi\sin\phi' = 0 \qquad (\sin\theta' \neq 0). \quad (6)$$

It is assumed that $\boldsymbol{m}$ is initially aligned with the y axis by a constant field $(\psi = \pi/2)$. The initial phase of ϕ' is therefore unimportant since $\boldsymbol{m}$ is precessing about the y axis with very small cone angle $(\theta' \simeq 0)$. The phase of ϕ' can be taken as that value which leads to the easiest solution of the differential equations, therefore the initial phase is chosen so that $\cos\phi'$ is exactly zero when $\dot\psi$ has its maximum value. Since $\dot\psi$ normally drops rapidly from its maximum value or else never attains a large value [compared with $\mu_0\gamma H_r(t)$] at all, it is permissible to neglect the $\dot\psi$ term in (5). For very small values of t the $\cot\theta'$ is quite large but the oscillatory nature of $\cos\phi'$ prevents any important contribution to $\dot\phi'$ from being made. The approximation is valid if $\dot\psi$ is never comparable to the precession frequency or if $\dot\psi$ approaches an impulse. Certain intermediate cases may require further consideration.

If the approximation is accepted it follows that

$$\dot\phi' \simeq -\gamma\mu_0 H_r(t) \quad (7)$$

$$\dot\theta \simeq -\dot\psi\sin\phi \quad (8)$$

and $\cos\phi' = 0$ when $\dot\psi = \dot\psi_{\max}$. If this maximum occurs when $t = u_0$ then

$$\phi' \simeq -\int_{u_0}^{t}\mu_0\gamma H_r(x)dx + \frac{\pi}{2}$$

and

$$\theta' \simeq -\int_0^t \dot\psi\cos\left[-\int_{cc_0}^{t}\mu_0\gamma H_r(x)dx\right]dt \quad (0 \leq t \leq T). \quad (9)$$

The value of θ' at the end of the transient $(t = T)$ is given by

$$\theta_T \simeq \int_0^T \dot\psi\cos\left[-\int_{u_0}^{t}\mu_0\gamma H_r(x)dx\right]dt. \quad (10)$$

Example

Consider a transient such that the magnitude of the pulsed field (H_0) is always constant and the angle ψ changes from ψ_0 to 0 at a constant rate in time T. Since $-\dot\psi = \psi_0/T$ and $H(x) = H_0$ it follows that

$$\theta_T \simeq \frac{\psi_0}{T}\int_0^T \cos\left[\int_{u0}^{t}(-\gamma\mu_0 H_0)dx\right]dt. \quad (11)$$

Since ψ has its maximum over the whole range of time $0 \leq t \leq T$ it appears that u_0 is ambiguous. However $\dot{\psi}$ cot θ' is a maximum when t is zero. Using this value and defining $\omega_0 = -\gamma\mu_0 H_0$

$$\theta_T \simeq \psi_0 \left|\frac{\sin \omega_0 T}{\omega_0 T}\right|. \quad (12)$$

It is apparent in this case that for θ_T to be a large fraction of ψ_0, T should be short compared with the period of precession $(2\pi/\omega_0)$, and not just the relaxation time.

Actually, because of the approximation involved this result is only strictly valid when $T \gg 0$ or when $\psi_0 \ll \omega_0 T$. Nevertheless, the rise time dependence is quite evident.

The example chosen is instructive but in practice T will never be short compared with the period of precession. The problem is not as hopeless as it seems, however, because it is possible to change ψ substantially in a small fraction of the rise time, if the pulsing geometry is properly chosen.

180 Degrees $-\alpha$ Pulse Field Geometry

Previous considerations have indicated that it is necessary to have $\psi(t)$ change very rapidly if θ_T is to be an appreciable angle. It is apparent that if the pulse field is applied at 180 degrees to the saturation field, $\psi(t)$ must be a step function independent of T. This implies that θ_T would be π radians and that the spins would be given the maximum amount of potential energy, independent of T (so long as T is short compared with the damping time constant). Unhappily this may not be the solution because of the following consideration. Consider layers of aligned spins which have suddenly been placed in an anti-parallel magnetic field. The equilibrium is unstable because of thermal agitation, and it is equally likely that the net moment vector from a given spin layer will want to move in any direction, so that on the average two layers or groups of spins want to precess 180 degrees out of phase, spiraling out and then in until both are aligned with the resultant field. Exchange forces prevent this, but spin wave modes are likely to build up quickly. The radiation fields from such spin waves cancel and all the potential energy initially given these spins must go into heating the lattice.

To overcome this difficulty the pulsed field may be oriented at 180 degrees$-\alpha$, where α is some small angle (see Fig. 3). The spins now have a preferred direction and want to precess in phase. However, as soon as α is nonzero, $\psi(t)$ departs from a true step function and θ_T again becomes a function of the rise time T, as well as the other parameters. If α is made very small the amount of this departure is not large but the spin wave buildup appears again nearly as pronounced as before since thermal agitation always spreads the spins randomly over some small cone angle. It appears that the smaller the value of α, the lower the temperature required for the same amount of spin wave buildup.

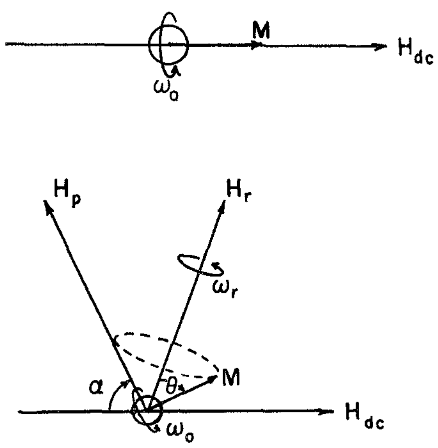

Fig. 3—180°$-\alpha$ pulsing geometry.

The magnetic field geometry previously discussed is shown in Fig. 4.

The pulsed magnetic field is assumed to be of the form as shown in Fig. 5.

The magnitude and phase of $H_r(t)$ are given by

$$|H_r(t)| = [H_p(t)^2 - 2H_{dc}H_p(t)\cos\alpha + H_{dc}^2]^{1/2} \quad (13)$$

and

$$\tan\psi = \frac{H_{dc}\sin\alpha}{H_p(t) - H_{dc}\cos\alpha}. \quad (14)$$

The latter expression leads to

$$-\dot{\psi} = \frac{H_{dc}\dot{H}_p(t)\sin\alpha}{H_p(t)^2 - 2H_p(t)H_{dc}\cos\alpha + H_{dc}^2} \quad (15)$$

over the region of interest $(0 < t < T)$, for which $H_p(t) = (H_0/T)t$.

The Larmor precession frequencies corresponding to H_0, H_{dc}, and H_r are $\omega_p = -\gamma\mu_0 H_0$, $\omega_{dc} = -\gamma\mu_0 H_{dc}$, and $\omega_r = -\gamma\mu_0 H_r$, respectively.

If two parameters, u_0 and v_0, are defined so that

$$u_0 = \frac{\omega_{dc}}{\omega_p} T\cos\alpha \quad (16)$$

and

$$v_0 = \frac{\omega_{dc}}{\omega_p} T\sin\alpha \quad (17)$$

then the integral (10), giving θ_T may be written as

$$\theta_T = \int_0^T \frac{V_0}{t^2 - 2u_0 t + u_0^2 + v_0^2} \cdot \cos\left[\frac{\omega_p}{T}\int_{u_0}^{t}[x^2 - 2u_0 x + u_0^2 + v_0^2]^{1/2}dx\right]dt. \quad (18)$$

Since $-\dot{\psi}$ consists of an impulse at $t = u_0$ when $\alpha \rightarrow 0$, the constant u_0 is evaluated so that $\cos\phi(u_0) = 0$, in accordance with previous remarks. The angle α has been assumed small, therefore a permissible approximation is to

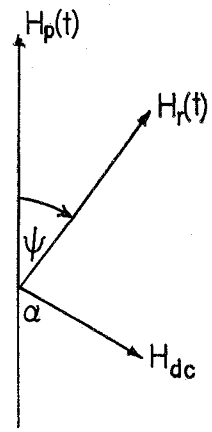

Fig. 4—Magnetic field geometry.

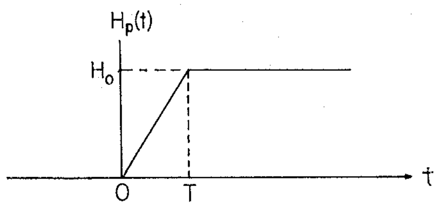

Fig. 5—Pulsed magnetic field as a function of time.

neglect the v_0^2 term in the $\int_{u_0}^{t}\omega(x)dx$ factor. The integral then reduces to

$$\theta_T \simeq \int_0^T \frac{v_0}{(t-u_0)^2+v_0^2}\cos\left[\frac{\omega_p}{2T}(t-u_0)^2\right]dt. \quad (19)$$

The first factor $(-\dot{\psi})$ in the integrand may be represented in the complex t plane $(t=u+iv)$ by a pair of poles as shown in Fig. 6.

In the limit as $\alpha \to 0$, v_0 vanishes and a double order pole exists at $t=u_0$. Since the integration proceeds along the real axis, it is clear that the major contribution to the integral occurs near $t=u_0$. The second factor in the integrand is unity at the same point and oscillates very rapidly for smaller or larger values of t. Fig. 7 represents the functions whose integrated product yields θ_T.

The cos $[\omega_p/2T(t-u_0)^2]$ factor varies so rapidly from plus to minus one that contributions to the integral are essentially zero except near $t=u_0$. If θ_T is to be large (it is $\pi-\alpha$ for $\alpha=0$ or $T=0$) it is clear that the resonant peak must have a width $(2v_0)$ comparable to the width (2Δ) of the positive half cycle of cos $[\omega_p/2T(t-u_0)^2]$, centered about $t=u_0$.

The evaluation of the integral (19) has been carried out numerically for two values of T (10^{-7} and 10^{-8} sec) for a range of values of α and ω_r. H_{dc} was assumed constant throughout and equal to approximately 500 gauss so that $\omega_{dc}=10^{10}$ rad/sec. The results are shown in Figs. 8 and 9.

Radiation Transient

When the pulse rise is completed there will exist a cone angle θ_T between the magnetization $\boldsymbol{m}$ and the re-

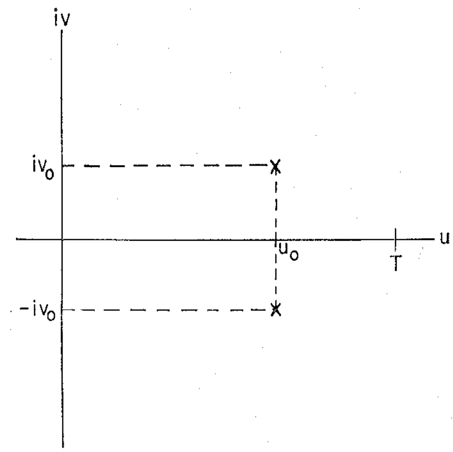

Fig. 6—Complex $(u+iv)$ plane.

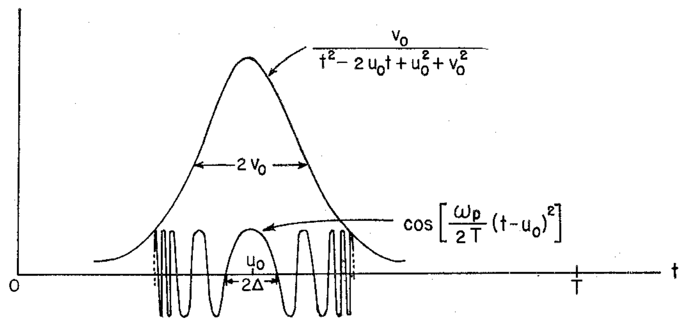

Fig. 7—Integrand functions.

sultant field $\boldsymbol{H}$ (which is now constant in magnitude and direction) $\boldsymbol{m}$ spirals into $\boldsymbol{H}$ at a precession frequency ω_r due to radiation and heat losses. If N_0 is the total number of spins available in the sample there is some fraction N_{eff}/N_0 capable of radiating (the remainder are assumed to be in the form of spin waves as discussed previously).

The radiation field is simply that due to a rotating magnetic moment[1] (or amperian loopcurrent) of magnitude m sin $\theta(t)$.

If α is large enough to ensure N_{eff}/N_0 being near unity, the initial cone angle, θ_T, will probably be relatively small (an engineering compromise is needed—as usual). Assuming a small angle for θ_T, it may be shown that the average radiated power per pulse transient of magnetic field is

$$P_{\substack{\text{ave. rad.}\\ \text{power per pulse}}} = K\frac{\mu_0 m^2}{12\pi c^3}N_{eff}^2\omega_r^4\theta_T^2 \quad (20)$$

where K is a coupling coefficient ($K=1$ if dipoles radiate to free space). The radiation time constant is given by

$$\tau_r = \frac{12\pi c^3}{KN_{eff}\,\mu_0 m\,|\gamma|\,\omega_r^3}. \quad (21)$$

The total time constant of the magnetization decay is

$$\tau_t = \frac{\tau_r\tau_l}{\tau_r+\tau_l} \quad (22)$$

[1] J. A. Stratton, "Electromagnetic Theory," McGraw-Hill Book Co., Inc., New York, N. Y., pp. 437–438; 1941.

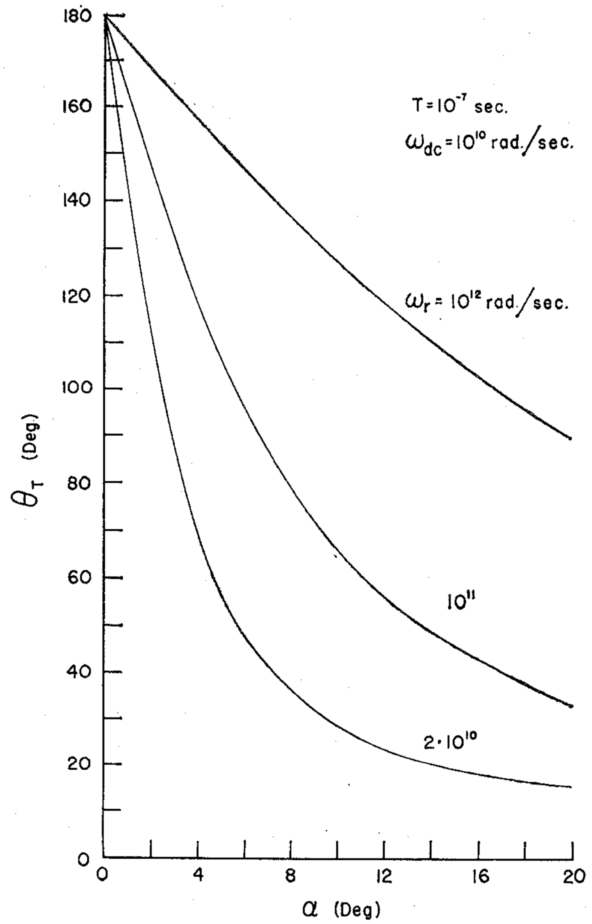

Fig. 8—θ_T vs α.

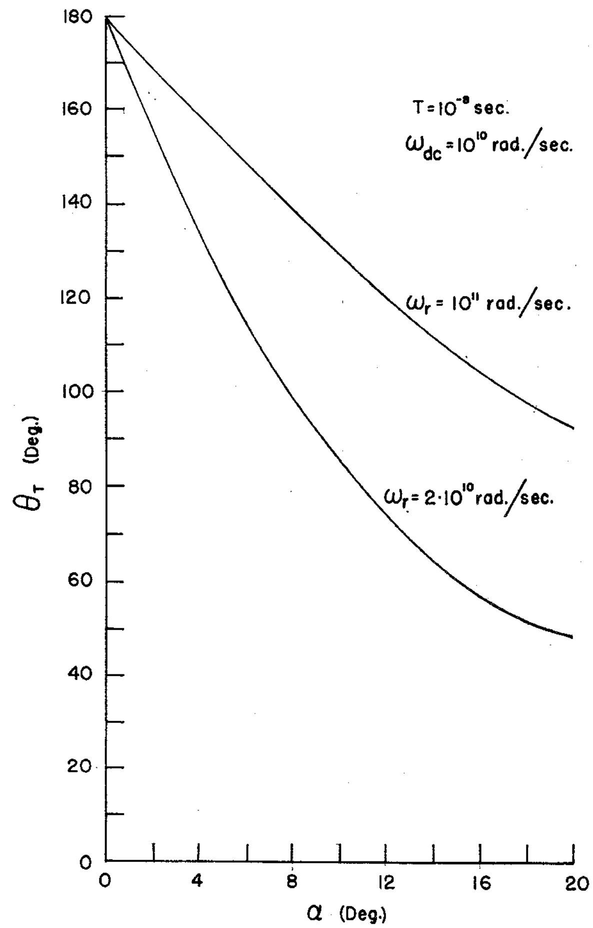

Fig. 9—θ_T vs α.

where τ_l is the lattice relaxation time assuming Bloch-Bloembergen damping.

The radiation efficiency is defined by and is equal to

$$\eta = \frac{\text{energy radiated}}{\text{energy coupled}} \simeq \frac{\dfrac{N_{eff}}{N_0}}{1 + \dfrac{\tau_r}{\tau_l}}. \quad (23)$$

It should be noted that a type of unfavorable instability appears possible. Assume that a small angle α is chosen and the initial operating temperature made low enough so that the spins are highly aligned to provide efficient operation. Now if in operation the crystal heats up a little, the spins spread out somewhat. This decreases the effective number of spins and increases the radiation time constant, thereby causing more power to be dissipated leading to a still further increase in temperature and decrease in efficiency. If the closed "loop gain" of the feedback system is greater than unity, the crystal eventually stops radiating and may even be destroyed.

Quantitative Considerations

The normal ferrites are impractical for such oscillators because the Bloch lattice relaxation time (τ_l) is very short ($10^{-8}-10^{-7}$ seconds) and it would be nearly impossible to provide large magnetic pulses with rise times (T) small compared with τ_l. The newly discovered yttrium iron garnet single crystals are reported to have intrinsic resonant line widths of the order of one oersted. This corresponds to a τ_l of $10^{-7}-10^{-6}$ seconds and a required T in the range $10^{-8}-10^{-7}$ seconds (possible to obtain for reasonable magnetic field strengths). The minimum saturation field (H_{dc}) required for these materials may be assumed to be $5/4\pi \times 10^5$ amperes per meter (500 gauss) which corresponds to an ω_{dc} of about 10^{10} rad/sec. The saturation magnetization, M_s, is approximately $17/4\pi \times 10^5$ amperes/meter ($4\pi M_s = 1700$ gauss) and there are 10^{19} spins/mm^3. It is instructive to compute the power and radiation efficiency.

For a one mm^3 crystal with a pulsed field of magnitude such that $\omega_r = 10^{11}$ rad/sec and $T = 10^{-7}$ seconds and assuming that $K \simeq \frac{1}{2}$, $\alpha \simeq 20$ degrees, $N_{eff} \simeq 3N_0/4$ and $\tau_l \approx 0.15$ μsec it follows that $\theta_T \simeq \pi/10$ radians. (The value of 36 degrees from the Fig. 8 curves is halved to take into account the fact that the pulse rise is not conservative.)

The average power per pulse is approximately 30 watts and the radiation efficiency 40 per cent. The pulse of radiation lasts about 0.05 μsec.

Conclusions

The preceding analysis has shown that under certain restrictive conditions the electron spins appear able to couple energy from a pulsed magnetic field and radiate it efficiently at some microwave frequency. The analysis has been done for a vastly simplified system where such factors as anisotropy have been neglected. Nevertheless, certain basic limitations such as the geometrical dependence of the initial precession cone angle (θ_T) have become apparent as well as the need to consider carefully the operating temperature.

Several alternate methods of coupling the magnetic energy from the pulsed field have been proposed by the author,[2] and depend on a pulsed RF field coupling energy to the spins before the spin wave spectrum can build up. This RF field would be of lower frequency than the radiation which is desired and so a pulsed magnetic field would raise the system to the desired energy level.

A great deal of fundamental knowledge must be obtained, and many engineering problems solved, before a practical oscillator can be built. With this goal in mind, experiments are being carried out at the Air Force Cambridge Research Center.

A last point worth mentioning is that the possibility of studying the relaxation behavior of the precessional cone angle of the magnetization by means of the observed radiation is an exciting idea which may lead to a more conclusive theory of ferrimagnetic resonance damping.

[2] Presented orally at the International Conference on Solid State Physics as Applied to Electronics and Telecommunications, Brussels, Belgium; June, 1958.

Acknowledgment

The author is deeply indebted to Prof. C. L. Hogan of Harvard University for bringing the subject to his attention, and for continued encouragement and criticism. Mrs. M. R. Hoes of Air Force Cambridge Research Center performed the numerical integrations.

Ferrite High-Power Effects in Waveguides*

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Summary—**Deterioration of ferrite devices caused by both high power thermal and nonlinear effects are discussed. It is shown that thermal effects can be described, at least qualitatively, by a simple exponential equation. A theoretical maximum power capacity is derived in terms of ferrite configuration parameters. The results of experiments with high peak powers at both *S*-band and *X*-band frequencies are compared with predictions of Suhl's theory on nonlinear, high power effects in ferrites. Steady-state and transient effects are considered. It is shown that high power effects may be eliminated in ferrite devices by properly choosing ferrite properties and geometry.**

Low POWER ferrite components generally deteriorate when subjected to high power.[1] This paper includes a discussion of the thermal and nonlinear[2] ferrite effects commonly found at high power, and a description of an anamolous transient effect.

Fig. 1 shows the drastic decrease with power in a ferrite resonance attenuation characteristic that, at low power levels, is effectively utilized to build isolators. Methods are indicated to eliminate these undesirable effects.

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[1] N. Sakiotis, H. N. Chait, and M. L. Kales, "Nonlinearity of propagation in ferrite media," Proc. IRE, vol. 43, p. 1011; August, 1955.

[2] H. Suhl, "The nonlinear behavior of ferrites at high microwave signal levels," Proc. IRE, vol. 44, pp. 1270–1284; October, 1956.

Thermal Effects

Assuming that the microwave power is absorbed at the top surface of the ferrite, and that the generated heat flows directly through the ferrite and into the waveguide wall, then a simple calculation can be made of the steady-state temperature distribution on the ferrite. As shown in Part A of the Appendix

$$T(X) - T_0 = \frac{0.24\alpha T}{WK} P_{\text{in}} e^{-\alpha X}$$

where

α = attenuation constant
$T(X)$ = temperature of the top surface of the ferrite (Fig. 2) at the longitudinal position x
K = thermal conductivity of the ferrite
W = width of the ferrite slab
t = thickness of the ferrite slab
T_0 = wall temperature.

In applying this equation, special care must be taken in establishing T_0; at high powers a large thermal gradient may exist through the cement between the ferrite and waveguide wall. Fig. 2 shows the theoretical temperature gradient in a specific ferrite. This gradient was qualitatively observed with a temperature indicator.